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THE NUMBER OF ZERO SOLUTIONS FOR COMPLEX CANONICAL DIFFERENTIAL EQUATION OF SECOND ORDER WITH CONSTANT COEFFICIENTS IN THE FIRST QUADRANT

Jelena Vujaković[1*], Miloje Rajović[2]

[1]Faculty of Sciences and Mathematics, University of Priština, Kosovska Mitrovica, Serbia.

[2]Faculty of Mechanical Engineering, University of Kragujevac, Kraljevo, Serbia.

ABSTRACT

The study of complex differential equations in recent years has opened up some of questions concerning the determination of the frequency of zero solutions, the distribution of zero, oscillation of the solution, asymptotic behavior, rank growth and so on. Besides, this is solved by only some classes of differential equations. In this paper, our aim was to determine the number of zeros and their arrangement in the first quadrant, for the complex canonical differential equation of the second order. The accuracy of our results, we illustrate with two examples.

Key words: Differential equations, function of frequency, sine solution, cosine solution, zero solutions.

1. INTRODUCTION

The study of complex differential equations, in terms Nevanlinna theory, becomes actual again since 1982, the publication of the following works: Bank (1988), (Bank & Laine, 1982; 1983), (Bank et al., 1989).

Then are generally treated canonical complex differential equations of second order with a coefficient which is an entire function. All of these studies have focused mainly on two general issues.

The first one involved the determination of the frequency of zero solutions, while the other studied the distribution and the asymptotic behavior of zero solutions in the first quadrant i.e., in the sector $0 \le \varphi \le \frac{\pi}{2}, |z| = R$. About the problem of distribution of zero solutions of complex differential equations, the case where the coefficient $a(z)$ is polynomial $P_n(z)$ is quite clear. When $a(z)$ is the transcendent function the situation is much more complex. Review of the scientific literature, such as (Gundersen, 1986), (Laine, 1993), (Shu Pei, 1994) and others, shows that there are mostly treated complex differential equations with transcendental coefficients e^z and coefficients derived from it: $e^z + P_n(z)$, $e^z \bullet P_n(z)$, $P_n(e^z)$, ... This is because in Nevanlinna theory as a measure of transcendence and infinite growth, takes the function e^z. Since $|e^z| = e^x, x \in \mathbb{R}$, as $x \to \infty$ function e^z tends to complex infinity of transcendent type.

Unlike classical Nevanlinna theory, we are using the the idea of (Dimitrovski & Mijatović, 1998), (Lekić et. al., 2012), (Vujaković et al., 2011), (Vujaković, 2012) developed a new approach in determining the location and number of zero solutions. This method looked better in the applications for us.

In this paper, the subject of our considerations is complex canonical differential equations of second order with constant coefficients.

2. PRELIMINARIES

For complex canonical differential equation of the second order :

$$\frac{d^2 w}{dz^2} + a(z) w(z) = 0 \qquad (1)$$

with an analytical coefficient $a(z) = \alpha(x, y) + i\beta(x, y)$, where $\alpha(x, y)$ and $\beta(x, y)$ are harmonic functions, by series-iterations method which are described in detail in the works (Dimitrovski & Mijatović, 1998), (Lekić et al., 2012), (Vujaković, 2012), we get two fundamental solutions: